MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2017/2018

PCL0016 - CALCULUS

(ALL groups)

6 JUNE 2018
9:00AM – 11:00AM
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This examination paper consists of **3 pages** (including the cover page) with **6 questions only**.

2. **Attempt ALL questions.**

3. Please write all your answers in the Answer Booklet provided. **Show all relevant steps** to obtain maximum marks.

Question 1

(a) Evaluate the following limits, if they exist.

(i) $\lim_{x\to 5} 8(x-1)(2-x)$ [2 marks]

(ii) $\lim_{x\to\infty} \dfrac{\sqrt{x^5+2}}{x^3+1}$ [3 marks]

(iii) $\lim_{\theta\to 0} \dfrac{\sin^3\theta}{\theta(1-\cos\theta)}$ [5 marks]

(b) Find the limit $\lim_{x\to 1}[f(x)-g(x)]$, given the following functions:

$f(x)=5$ and $g(x)=\dfrac{-2x^2+x+1}{1-x}$ [5 marks]

(c) For what values of a is $h(x)$ continuous at every x?

$$h(x)=\begin{cases} a^2x^3+ax^2 & , \quad x<1 \\ x+5 & , \quad x\geq 1 \end{cases}$$ [5 marks]

Question 2

(a) Differentiate $y=\dfrac{5x+\sqrt{x}}{3x^2+1}$ and simplify your answer. [5 marks]

(b) Given $x\cos(y)+x^2=1$. Find $\dfrac{dy}{dx}$ using implicit differentiation. [4 marks]

(c) Use local linear approximation to estimate the value of $\sqrt[3]{126}$. [6 marks]

Question 3

A function $f(x)$ is denoted by $f(x)=-x^2(x-1)(x+1)$.

(a) Find the x-intercept(s) and y- intercept(s) of the graph of f, if any. [2 marks]

(b) Find the interval(s) on which $f(x)$ is increasing and/or decreasing. [4 marks]

(c) Find the interval(s) on which $f(x)$ is concave up and/or down. [3 marks]

(d) Identify the relative extrema of $f(x)$. [3 marks]

(e) Sketch the graph of $f(x)$. [3 marks]

Question 4

(a) Find:

(i) $\int \cos(x+1)\left(2+\sin(x+1)\right)^{100}\,dx$ using integration by substitution.

[3 marks]

(ii) $\int \frac{x}{e^{2x}}dx$ using integration by parts.

[3 marks]

(iii) $\int_3^4 \frac{\ln(5x-5)}{x-1}dx$ using definite integration by substitution. [3 marks]

(b) Given the rational function

$$f(x)=\frac{16}{x^3-2x^2},$$

use partial fraction decomposition to show that it can be written in the form

$$f(x)=-\frac{A}{x}-\frac{B}{x^2}+\frac{C}{x-2}$$ [4 marks]

Then, determine $\int f(x)dx$.

[2 marks]

Question 5

A region R is bounded by the curve $y=\sqrt{x}$, the line $y=3$ and the y-axis.

(a) Sketch R. [4 marks]

(b) Calculate the area of R. [5 marks]

(c) Find the volume of the solid generated when R is revolved about the x- axis using the method of washers. [6 marks]

Question 6

(a) Solve $\frac{dy}{dx}=e^{-3y}x^2$ with initial condition $y(2)=0$. [6 marks]

(b) Solve the differential equations below:

(i) $y''+y'-6y=0$, given $y(0)=0$ and $y'(0)=10$. [7 marks]

(ii) $y''+9y=0$, given $y(0)=2$ and $y'(\frac{\pi}{3})=-3$. [7 marks]

End of Paper